RADIATIVE HEAT EXCHANGE BETWEEN TWO ZONES, ONE OF WHICH CONSISTS OF SUSPENDED PARTICLES

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Zonal calculation of radiative heat exchange is examined for a closed system which consists of an aggregate of suspended gray particles and of gray shell [casing] walls.

In contemporary thermal engineering we frequently encounter high-temperature processes of heat and mass transfer in which the heat carrier is represented by a mixture of a gas or liquid containing suspended solid particles. Only slight attention has been devoted to the problem of caculating radiative heat exchange in such systems, and then usually is limited to the treatment of the heat carrier as a uniform semitransparent body with effective physical characteristics.

However, in certain cases it may prove advisable to give some consideration to the individual physical properties of the discrete particles. This aim can be achieved for a system with several particles by the usual zonal method of calculating radiative heat exchange. However, with a large number of particles this approach is impossible since approximation of integral equations of radiation results in a tremendous number of algebraic equations. On the other hand, it may be assumed that with a large number of suspended particles present in the system, their location and distribution through the volume will be defined by the laws of probability and aerodynamics. If the aerodynamic conditions are identical throughout the entire volume, the probable distribution of the particles will be uniform. The processes of radiant-energy transfer in a cloud of absolute-blackbody particles suspended in a transparent medium have been dealt with in many papers. For example, [1] presents a formula for the emissivity (absorption coefficient) of a dust medium in the following form:

$$1-\exp\left(-\frac{F_1 l}{4V}\right)=1-\exp(-kl). \qquad (1)$$

Here it is assumed that the particles are rather large in size, so that wave phenomena relative to the particles need not be taken into consideration. This assumption is not valid for fine dust and an experimental value for the coefficient k is therefore introduced into the calculation, as well as to account for the actual physical properties of the particle surfaces. The dust cloud in this case is regarded as a continuous medium. However, (1) is the limit case of the formula which is derived for a system of discrete uniformly distributed particles [1] and it is therefore useful also in the calculation of the radiative heat exchange of a system with discrete particles, if the latter are present in extremely large numbers.

To derive the formulas of the resulting radiation in the two-zone variant, we have also made the following assumptions: 1) all bodies in the system are "gray," and the suspension medium is transparent; 2) all particles exhibit the identical temperature T_1 and an identical absorption coefficient a_1 for the surfaces, with the casing walls characterized by corresponding constants T_2 and a_2; 3) we are considering a closed system.

According to the cited assumptions, the system is divisible into two zones: zone 1 is an accumulation of particles having a common surface F_1 and temperature T_1, while zone 2 is a shell [casing] with a common surface F_2 and temperature T_2.

When dealing with three-dimensional radiation we employ the concept of effective radiation-layer thickness l_{ef}, defined by the formula

$$l_{ef}=ml_0=m\frac{4V}{F_2}. \qquad (2)$$

Comparison of (1) and (2) shows that

$$F_1/F_2=kl_0. \qquad (3)$$

Consequently, the emissivity a_V of an absolute-blackbody cloud may be defined in the form of a function of kl_0, if the particle cloud is regarded as a continuous medium, or in the form of a function of F_1/F_2, if the collection of discrete particles is regarded as a zone having a common surface F_1:

$$a_V=1-\exp(-mkl_0)=1-\exp\left(-m\frac{F_1}{F_2}\right). \qquad (4)$$

Equations (3) and (4) may be used to derive the equations of the mutual surfaces h_{ki} and the average angular coefficients φ_{ki} of direct radiative heat exchange, i.e., without consideration of multiple reflections.

Indeed, the direct radiative exchange of energy between zones 1 and 2 is proportional to a_V, i.e.,

$$h_{12}=h_{21}=a_V F_2, \qquad (5)$$

and according to the closure property we have

$$h_{11}=F_1-h_{12}=(kl_0-a_V)F_2, \qquad (6)$$

$$h_{22}=F_2-h_{21}=(1-a_V)F_2. \qquad (7)$$

The matrix of the angular coefficients is consequently defined in the form

$$\begin{pmatrix} \varphi_{11} & \varphi_{21} \\ \varphi_{12} & \varphi_{22} \end{pmatrix} \equiv \begin{pmatrix} 1-\frac{a_V}{kl_0} & a_V \\ \frac{a_V}{kl_0} & 1-a_V \end{pmatrix}. \quad (8)$$

The formula of the resulting radiant flux in a two-zone closed system, with consideration of repeated reflection of the radiant fluxes, is cited in the literature [1, 2] and has the following form:

$$Q=\sigma_0 h_{12}\left(T_1^4-T_2^4\right) \Big/ \left[1+\varphi_{12}\left(\frac{1}{a_1}-1\right)+\right.$$

$$\left.+\varphi_{21}\left(\frac{1}{a_2}-1\right)\right]. \quad (9)$$

In the subject case the theoretical formula is derived directly from (8) which according to (5)–(8) assumes the following form:

$$q \equiv Q/\sigma_0\left(T_1^4-T_2^4\right)=$$

$$=F_1 \Big/ \left[\frac{kl_0}{a_V}+\left(\frac{1}{a_1}-1\right)+kl_0\left(\frac{1}{a_2}-1\right)\right]=$$

$$=F_2 \Big/ \left[\frac{1}{a_V}+\frac{1}{kl_0}\left(\frac{1}{a_1}-1\right)+\left(\frac{1}{a_2}-1\right)\right]. \quad (10)$$

It is not difficult to see that the above-cited principles may be extended to more complex systems with a larger number of zones. Here the increase in the number of volume zones can be achieved by dividing the space into individual volumes V_i characterized by a total particle surface F_{1i}. On the other hand, we may examine several groups of various particles with corresponding total surfaces F_{1i} occupying the same volume V. We note that the principal radiation of a "gray" gas may be calculated also from (10), if according to (2) and (3) we conditionally assume $F_1 = 4kV$.

Application of (10) to gaseous and dust media for which it is assumed that $a_1 = 1$, while the coefficient of beam attenuation k is determined from experimental data, if we neglect the effect of scattering, yields

$$q=F_2 \Big/ \left[\frac{1}{a_V}+\frac{1}{a_2}-1\right]. \quad (11)$$

Formula (11) is usually presented in the literature for a plane-parallel layer or for one-dimensional schemes of radiative heat exchange; however, it follows from the foregoing that it is also suitable for volumes of any geometric shape.

The exchange of heat between an aggregate of suspended "gray" particles and the shell is determined by their temperatures, their absorption coefficients, and by the total surfaces:

$$q=F_1 \Big/ \left[\frac{F_1}{F_2}\frac{1}{a_V}+\left(\frac{1}{a_1}-1\right)+\right.$$

$$\left.+\frac{F_1}{F_2}\left(\frac{1}{a_2}-1\right)\right]. \quad (12)$$

For small concentrations of particles ($F_1/F_2 < 0.5$) we can limit ourselves to the first terms in the series expansion of a_V and bring (12) to the following form:

$$q=F_1 \Big/ \left[\frac{1}{a_1}+\frac{F_1}{F_2}\left(\frac{1}{a_2}-1\right)+f\left(\frac{F_1}{F_2}\right)\right], \quad (13)$$

where $f(F_1/F_2)$ depends on the volume shape. For example, for a sphere, $f(F_1/F_2) = (9/16)(F_1/F_2)$; for a cylindrical volume, $f(F_1/F_2) = (2/3)(F_1/F_2)$; for a plane-parallel layer, $f(F_1/F_2) = [0.404 - 0.25 \ln(F_1/F_2)]\, F_1/F_2$.

Example. We have a heat exchanger in the form of a long vertical tube with an inside diameter D = 40 mm through which spherical particles suspended in air are moving and are being heated; these spheres have diameters of d = 0.6 mm. The volume concentration of the particles is $\mu_V = 10^{-3}\ m^3/m^3$. The absorption factors for the particle and tube-wall surfaces are, respectively, $a_1 = 0.8$ and $a_2 = 0.67$. We will carry out the calculation over 1 m of tube length.

The total particle surface is

$$F_1=\frac{6}{d}\mu_V\frac{\pi D^2}{4}=\frac{6\cdot 10^3\cdot 10^{-3}\pi\, 40^2\cdot 10^{-6}}{0.6\cdot 4}=$$

$$=0.0126\ \text{m}^2.$$

The tube side-wall surface is

$$F_2=\pi D=\pi 40\cdot 10^{-3}=0.126\ \text{m}^2.$$

The ratio $F_1/F_2 = 0.1$.
Since $F_1/F_2 < 0.5$, from (13) we have

$$q=\frac{F_1}{\frac{1}{a_1}+\frac{F_1}{F_2}\left(\frac{1}{a_2}-1\right)+\frac{2}{3}\frac{F_1}{F_2}}=$$

$$=\frac{0.0126}{1.25+0.1\cdot 0.5+0.1\cdot 0.67}=0.00922\ \text{m}^2.$$

NOTATION

a and a_V are the emissivities of the volume; D and d are the diameters of the tube and particle, respectively; F is the over-all geometrical surface of the zone; h_{ki} is the mutual radiation-exchange surface; k is the attenuation coefficient; l is the radiative layer thickness; l_0 is the characteristic dimension of the radiating system; Q is the net radiative flux; q is the net reduced radiative flux; T is the absolute temperature; V is the volume of the radiative system; μ_V is the volume fraction of particles; σ_0 is the Stefan–Boltzmann constant; φ_{ki} is the mean angular coefficient of direct radiant heat transfer. Subscripts: 1) particles; 2) walls of casing.

REFERENCES

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